International Journal of Theoretical Physics, Vol. 15, No. 7 (1976), pp. 513-520

A New Particle Solution in Aesthetic Field Theory

MURRAY MURASKIN and BEATRICE RING

Stanford Linear Accelerator Center, Stanford, California,

and

Department of Physics, University of North Dakota, Grand Forks, North Dakota

Received: 18 *July* 1975

Abstract

In present theories a particle is commonly associated with a singularity of the field. A more realistic picture would describe the particle by an intense but singularity-free field. We have found a new solution to the aesthetic field equations for which the field associated with the particle has a very large magnitude. The particle appears to be bounded despite the large numbers appearing in the solution. We prove that this present solution is not equivalent to the $O(3)$-invariant solution discussed in Muraskin (1973b). Since our present solution appears well-behaved, the suggestion is that we do not confine ourselves to $O(3)$-invariant data in future work. Owing to the large magnitude fields, we were unable to study trajectories of the particle in any detail. There is nothing wrong, in itself, with large numbers. The present solution, which we have now studied, is the first instance in our work on aesthetic field theory in which large numbers appear without the suggestion of unboundedness.

1. *Introduction*

There are some mathematical principles that are of such an attractive nature that they would be expected to be incorporated in physics at a fundamental level. We have been studying several principles which we may reasonably believe to fit in this category

The Dirac equation is an equation that treats all first derivatives in a uniform way. This can be considered an "aesthetic" principle. The question to which we have addressed ourselves is whether or not it is possible to formulate a theory where all derivatives as well as all tensors are treated in a uniform way. We have shown, in previous work (Muraskin, 1970, 1975) that it is possible to do so. The field theory assumes the existence of a hierarchy of tensors in a Cartesian space (together with a time axis). We then hypothesize the existence of a universal change function that determines the change of all tensor functions. Since the change function is, itself, a tensor function, Aristotelian logic demands that the change function must determine its own change as well. This leads to the field equations $\Gamma^i_{jk;l=0}$. Such a theory has, we have found in previous papers, considerable content. Furthermore, it is difficult to believe that solutions with even greater complexity do not exist. We shall continue our study of the solutions of these equations in this paper.

2. *Discussion of O(3) Invariance*

There are some reasons to question an underlying structure invariant under $O(3)$ from a conceptual point of view. Such invariance was made use of in Muraskin (1973a, 1973b).

(a) If $\Gamma^\alpha_{\beta\gamma}$ is a function of x as in the case of $e^\alpha{}_i \to \delta^\alpha_i$ at infinity, then $\Gamma^\alpha_{\beta\gamma}$ is really not a basic object in the theory. We recall

$$\Gamma^i_{jk} = e_\alpha{}^i e^\beta{}_j e^\gamma{}_k \Gamma^\alpha_{\beta\gamma} \tag{2.1}$$

Thus, if $e^\alpha{}_i \to \delta^\alpha_i$ at infinity, then Γ^i_{jk} cannot go to zero there, unless $\Gamma^\alpha_{\beta\gamma}$ is a function of x. Since Γ^i_{jk} is not invariant under $O(3)$, even in Muraskin (1973b), one can question why $\Gamma^\alpha_{\beta\gamma}(x)$ should be. In Λ_{ijk} theory (Muraskin, 1973a) $\Lambda_{\beta\gamma}$ was constant, and thus it is a basic object in the theory. In this case, the group invariance is a more natural hypothesis.

(b) Invariance of $\Gamma^\alpha_{\beta\gamma}$ under $O(3)$ may be too strong a restriction. That is, the data we shall present, in Section 3, appear to lead to a bounded particle. If we rotate the coordinate system, we have

$$\Gamma'^{\,i}_{jk} = a_s{}^i a^m{}_j a^n{}_k \Gamma^s_{mn} \tag{2.2}$$

In this paper, we shall study, in particular, the component Γ^1_{11}. Using the Γ'^i_{jk}, we get a similar looking bounded particle for Γ'^1_{11}. This suggests that the requirement of $O(3)$ invariance of $\Gamma^\alpha_{\beta\gamma}$ does not appear to be necessary in order to have a system that does not have its structure altered by rotations. These arguments suggest that we study the non-$O(3)$-invariant situation.

3. *A New Set of Data*

We consider the following $\Gamma^\alpha_{\beta\gamma}$:

$\Gamma^1_{11} = 0.1$	$\Gamma^1_{12} = 0$	$\Gamma^1_{13} = 0$	$\Gamma^1_{10} = 0.1$
$\Gamma^1_{21} = 0.1$	$\Gamma^1_{22} = 0$	$\Gamma^1_{23} = 0$	$\Gamma^1_{20} = 0$
$\Gamma^1_{31} = 0.1$	$\Gamma^1_{32} = 0$	$\Gamma^1_{33} = 0$	$\Gamma^1_{30} = 0$
$\Gamma^1_{01} = 0.1$	$\Gamma^1_{02} = 0$	$\Gamma^1_{03} = 0$	$\Gamma^1_{00} = 0$
$\Gamma^2_{11} = 0$	$\Gamma^2_{12} = 0.1$	$\Gamma^2_{13} = 0$	$\Gamma^2_{10} = 0$

$$
\begin{array}{llll}
\Gamma^2_{21} = 0 & \Gamma^2_{22} = 0.1 & \Gamma^2_{23} = 0 & \Gamma^2_{20} = 0.1 \\
\Gamma^2_{31} = 0 & \Gamma^2_{32} = 0.1 & \Gamma^2_{33} = 0 & \Gamma^2_{30} = 0 \\
\Gamma^2_{01} = 0 & \Gamma^2_{02} = 0.1 & \Gamma^2_{03} = 0 & \Gamma^2_{00} = 0 \\
\Gamma^3_{11} = 0 & \Gamma^3_{12} = 0 & \Gamma^3_{13} = 0.1 & \Gamma^3_{10} = 0 \\
\Gamma^3_{21} = 0 & \Gamma^3_{22} = 0 & \Gamma^3_{23} = 0.1 & \Gamma^3_{20} = 0 \\
\Gamma^3_{31} = 0 & \Gamma^3_{32} = 0 & \Gamma^3_{33} = 0.1 & \Gamma^3_{30} = 0.1 \\
\Gamma^3_{01} = 0 & \Gamma^3_{02} = 0 & \Gamma^3_{03} = 0.1 & \Gamma^3_{00} = 0 \\
\Gamma^0_{11} = -0.1 & \Gamma^0_{12} = -0.1 & \Gamma^0_{13} = -0.1 & \Gamma^0_{10} = 0 \\
\Gamma^0_{21} = -0.1 & \Gamma^0_{22} = -0.1 & \Gamma^0_{23} = -0.1 & \Gamma^0_{20} = 0 \\
\Gamma^0_{31} = -0.1 & \Gamma^0_{32} = -0.1 & \Gamma^0_{33} = -0.1 & \Gamma^0_{30} = 0 \\
\Gamma^0_{01} = 0 & \Gamma^0_{02} = 0 & \Gamma^0_{03} = 0 & \Gamma^0_{00} = 0.1
\end{array}
\tag{3.1}
$$

We take for $e^{\alpha}{}_i$, the following

$$
\begin{array}{llll}
e^1{}_1 = 0.88 & e^1{}_2 = -0.42 & e^1{}_3 = -0.32 & e^1{}_0 = 0.22 \\
e^2{}_1 = 0.5 & e^2{}_2 = 0.9 & e^2{}_3 = -0.425 & e^2{}_0 = 0.3 \\
e^3{}_1 = 0.2 & e^3{}_2 = -0.55 & e^3{}_3 = 0.89 & e^3{}_0 = 0.6 \\
e^0{}_1 = 0.44 & e^0{}_2 = -0.16 & e^0{}_3 = 0.39 & e^0{}_0 = 1.01
\end{array}
\tag{3.2}
$$

An interesting observation is that these data satisfy the integrability equations with $R^i{}_{jkl} \neq 0$ but does not satisfy the g_{ij} integrability equations, at least for a diagonal g_{ij}. The g_{ij} integrability equation has

$$g_{th}R^t{}_{imk} + g_{it}R^t{}_{hmk} = 0 \tag{3.3}$$

Using

$$
\begin{aligned}
\Gamma^i_{jk} &= e_{\alpha}{}^i e^{\beta}{}_j e^{\gamma}{}_k \Gamma^{\alpha}_{\beta\gamma} \\
g_{ij} &= e^{\alpha}{}_i e^{\beta}{}_j g_{\alpha\beta}
\end{aligned}
\tag{3.4}
$$

with an arbitrary $e^{\alpha}{}_i$ (possessing an inverse, at least at the origin) we get

$$g_{\lambda\alpha}R^{\lambda}{}_{\beta\gamma\delta} + g_{\beta\lambda}R^{\lambda}{}_{\alpha\gamma\delta} = 0 \tag{3.5}$$

For $\alpha = 1$, $\beta = 1$, $\gamma = 2$, $\delta = 1$ we get for a diagonal $g_{\alpha\beta}$

$$g_{11}R^1{}_{121} = 0 \tag{3.6}$$

But $R^1{}_{121}$ is not zero for the $\Gamma^{\alpha}_{\beta\gamma}$ we have introduced. Thus, the integrability equations for g_{ij} cannot be satisfied for diagonal $g_{\alpha\beta}$. There exists, however, nondiagonal symmetric second-rank tensors for which the g_{ij} integrability equations can be satisfied. For example, we may take $g_{ij} = \Gamma^t_{ti}\Gamma^s_{sj}$. Here, g_{ij} is constructed from products of Γ^i_{jk}.

This situation serves to introduce another variant within aesthetic field theory that we have not needed before this. In our earlier papers, we have assigned $\Gamma^{\alpha}_{\beta\gamma}$ and $g_{\alpha\beta}$ in an arbitrary fashion at the origin point. But, so far as the field equations are concerned, g_{ij} is a secondary type field. That is, Γ^{i}_{jk} determines the change of g_{ij} but g_{ij} does not effect the change of Γ^{i}_{jk}. Thus, a simplifying hypothesis would be to consider only Γ^{i}_{jk} as arbitrary at the origin point.

Let us return to studying the properties of the new set of data. We see that it contains the same components that appear in the $O(3)$-invariant data, namely,[1]

$$\Gamma^{1}_{10} = \Gamma^{2}_{20} = \Gamma^{3}_{30} = \Gamma^{1}_{01} = \Gamma^{2}_{02} = \Gamma^{3}_{03} = \Gamma^{0}_{00} = -\Gamma^{0}_{11} = -\Gamma^{0}_{22} = -\Gamma^{0}_{33} \quad (3.7)$$

But, in addition, we have components not present in the $O(3)$ case.

The present data cannot be written in the form

$$\Gamma^{\alpha}_{\beta\gamma} = \delta^{\alpha}_{\beta}\phi_{\gamma} + \delta^{\alpha}_{\gamma}\theta_{\beta} + \psi^{\alpha}g_{\beta\gamma} + g^{\alpha\rho}B^{\lambda}\epsilon_{\lambda\rho\beta\gamma} \quad (3.8)$$

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The $O(3)$ data do have such a structure. In the $O(3)$ case, Γ^{t}_{tk} is parallel to Γ^{t}_{kt} as well, and this result is maintained by the field equations. In the present data, Γ^{t}_{tk} is not parallel to Γ^{t}_{kt}.

We note, it is not possible to obtain the present data from $O(3)$-invariant data by means of an $e^{\alpha}{}_{i}$ transformation (such that the inverse $e_{\alpha}{}^{i}$ exists). We see this very simply as follows: given $\Gamma^{\alpha}_{\alpha\beta} = \Gamma^{\alpha}_{\beta\alpha}$ we get from

$$\Gamma^{\alpha}_{\beta\gamma} = e^{\alpha}{}_{i}\, e_{\beta}^{j}\, e_{\gamma}{}^{k}\Gamma^{i}_{jk} \quad (3.9)$$

$$e_{\gamma}{}^{k}(\Gamma^{t}_{tk} - \Gamma^{t}_{kt}) = 0 \quad (3.10)$$

It then follows on multiplication by $e^{\gamma}{}_{m}$ that $\Gamma^{t}_{tk} = \Gamma^{t}_{kt}$. Thus, by means of an $e^{\alpha}{}_{i}$ transformation, we cannot get $\Gamma^{t}_{tk} \neq \Gamma^{t}_{kt}$ starting with $O(3)$-invariant data. This means that the present data are not equivalent to our $O(3)$-invariant data. In Muraskin (1974), we had another set of data, that was not quite the same as our $O(3)$-invariant data, which led to a bounded particle. But in this case, we cannot use the argument given above since Γ^{t}_{tk} was the same as Γ^{t}_{kt} there.[2] We have not, as yet, proved whether these data are equivalent to $O(3)$ data.

[1] The B^0 terms Γ^{1}_{23}, Γ^{1}_{32}, Γ^{2}_{13}, Γ^{2}_{31}, Γ^{3}_{21}, Γ^{3}_{12} are not present in (3.1). However, we showed, in Muraskin (1974), that these components are not necessary in obtaining a bounded particle.

[2] $\Gamma^{t}_{tk} = \Gamma^{t}_{kt}$ appears also in Muraskin (1975).

We conclude then, that data (3.1) are not equivalent to our $O(3)$-invariant data owing to important differences in structure.[3]

In the next section we discuss the results of our computer studies for the new data.

4. *Computer Results*

Although we have not imposed the condition that any of the components be a maximum or minimum at the origin, nevertheless, we have found both a planar maximum and minimum close to the origin. In this respect, our results are similar to data 3 of Muraskin (1974). The latter data are of the $O(3)$-invariant type.

The contour lines surrounding the maximum and minimum for Γ^1_{11} in the x–y plane have a shape resembling a series of ellipses with minor axis much smaller than the major axis. Again, the results are much the same as in the $O(3)$ work (Muraskin, 1973b).[4] We did most of our mapping close to the planar maximum and minimum in the x–y plane. We do not expect the x–y or y–z planes to be fundamentally very different owing to the symmetric way that the indices 1, 2, 3 appear in $\Gamma^\alpha_{\beta\gamma}$.

The big difference with our previous work, that we have uncovered, has to do with the magnitude of the field components. We ran a comparison run by taking the $O(3)$-invariant data in Muraskin (1973b), but with parameters at the origin taken to be the same as (3.1) and (3.2). The only difference then in the two sets of data, is the non-$O(3)$-invariant components. That is, except for the presence of Γ^1_{21}, Γ^1_{31}, Γ^2_{12}, Γ^2_{32}, Γ^3_{13}, Γ^3_{23}, Γ^0_{21}, Γ^0_{13}, Γ^0_{21}, Γ^0_{23}, Γ^0_{32}, Γ^0_{31} and the absence of Γ^1_{23}, Γ^2_{13}, Γ^2_{31}, Γ^1_{32}, Γ^3_{12}, Γ^3_{21} the data would be the same.

For the $O(3)$-invariant data we obtained a maximum for Γ^1_{11} in the x–y plane with the value 0.27. On the other hand, for the noninvariant data the maximum was in excess of 0.17×10^7 in the x–y plane. We might add that the components of Γ^i_{jk} were very large at this point. We did not make a serious attempt at getting the exact value of the maximum since this would have required too much computer time. Also, no attempt was made to find the maximum in three-dimensional space. We could expect this maximum to be much greater than the 0.17×10^7 figure. Thus, we have obtained an interesting property of the new particle system, which was not observed in our previous work. The maximum has an extremely large magnitude.[5]

Can we be sure that a singularity is not developing? We cannot offer any proof one way or the other. However, we have surrounded the maximum in the x–y plane. This suggests that we should be able to surround the maximum

[3] Under a rotation of coordinates, $\Gamma^\alpha_{\beta\gamma}$ is altered, but any components that were zero to start with do not become nonzero under the rotation.

[4] In Muraskin (1973b), we studied the behavior of g_{00}.

[5] It has not been proved that this new effect cannot be obtained for $O(3)$-invariant data for some choice of data at the origin. However, the comparison run discussed above seems to suggest a role for the noninvariant components.

in a three-dimensional plot, as x, y, z are all equivalent in $\Gamma^{\alpha}_{\beta\gamma}$. We have been able to cross the particle on the line $y = -71.17$, which is quite close to where we think the maximum in the x–y plane is located. We did obtain a turnabout point when we did this. The values of Γ^1_{11} as we approach the planar maximum are given below[6]:

y	x	Γ^1_{11}
−68	−10.0	0.34
−68	−9.0	0.43
−68	−8.0	0.57
−68	−7.0	0.82
−68	−6.0	1.3
−68	−5.0	2.6
−70	−4.0	11.7
−71	−4.0	14.2
−71	−3.3	369
−71.17	−3.209	0.3×10^4
−71.17	−3.152	0.17×10^7
−71.17	−3.132	0.2×10^5
−71.17	−3.072	0.15×10^4
−71.17	−2.84	96
−71	−2.0	6.6
−71	−1.0	1.7
−71	0	0.67

We note the extremely sharp rise for Γ^1_{11} close to the maximum.

Is such a large-magnitude maximum a good or bad thing? We would say that the present particle is somewhat more realistic than our previous results. That is, the particles in nature obey Coulomb's law and Newton's gravitational law. Here, the field goes to infinity at the location of the particle. Although this infinity would be expected to be modified by other factors, we would nevertheless expect the field to be quite large at the location of the particle.

The minimum in the x–y plane was found to be roughly at $x = -12.38$, $y = -221.12$. Here, $\Gamma^1_{11} = -0.0989$. We tried to find the minimum in three dimensions. However, large distances from the origin are involved, and again we ran into practical limitations involving computer time. We hunted for the minimum point as far as $x = -11, y = -1107, z = -400$.

We made long runs down the coordinate axis. As in our previous work (Muraskin, 1973), the field components got small if we go far enough from the origin. This is consistent with the notion of natural boundary conditions. However, due to the large distance involved in looking for the minimum and large magnitudes involved in looking for the maximum, we have not been able to be as extensive as in our previous computer work. As a con-

6 These values have been obtained by approaching the planar maximum in two directions. The grid used was also rather coarse.

sequence of this, our conclusions are inferred from planar maps at $z = 0$ and from runs down the coordinate axes (including the time axis).

To summarize, we have obtained the following features in the present work: (1) We have found a two-particle system similar to previous two-particle systems, with an important difference: The magnitude of the maxima is now many orders of magnitude larger than previously. (2) We can prove that the present data are not equivalent to our previous $O(3)$-invariant data.

5. *Discussion*

We recount, here, some features of the present version of the theory: (1) The change function is the basic field. (2) Other fields can be constructed from the change function by contraction, forming tensor products, and differentiation. All such fields have their changes treated by a uniform procedure. Thus, all higher derivatives and all tensor fields are treated in a uniform way by the theory. (3) The change function is determined by this same prescription. This leads to the field equations.

6. *Outlook*

After studying symmetry solutions of the integrability equations (Muraskin 1973a, 1973b, 1974) we made the following statement (Muraskin and Ring, 1972): "The basic problem, we feel, is to find general solutions of the integrability equations. Unfortunately, this is rather complicated." It should be said though, that much has been accomplished since that paper. We now have observed the following in our computer studies: (1) particle-like solutions, some involving collisions of two particles; (2) no sign of singularities appearing; (3) fields approaching zero outside the particle—thus, a damping mechanism appears to be at work; (4) an indefinite number of turnabout points can be made to appear along an axis (Muraskin 1974) (although, in this case, we have not been able to satisfy integrability); (5) solutions having the property of unboundedness. We would say that these ingredients are the kind one needs to build an interesting universe. What we need is a set of data that blends in the various characteristics above (the damping effect should dominate the unbounded tendency).

We have already embarked on such a program. By assuming that $\Gamma^{\alpha}_{\beta\gamma}$ has the values 0.1, −0.1, or 0, we have already obtained some 200 solutions of the integrability equations. A set of $O(3)$-invariant data has been obtained by this procedure. The data used in this paper have also been obtained using this method. The project of permuting the values ±0.1 and 0 for $\Gamma^{\alpha}_{\beta\gamma}$ is a lengthy program[7] for which we have completed only certain limited phases. Dr. Elizabeth Cuthill of the Naval Ship Research Laboratory in Washington D.C., using IAM (FORMAC or REDUCE can also be used) has been able to

[7] We have been studying, in particular, the case for which the $\Gamma^{\alpha}_{\beta\gamma}$ components are unchanged under 1, 2, 3 cyclic permutation.

solve the algebraic integrability equations appearing in Muraskin (1971) and has verified the answers appearing there. The algebraic approach using the computer is still a very formidable problem considering the complexity of the integrability equations. However, we feel an attempt should be made in this direction.

Acknowledgment

We would like to thank Professor S. D. Drell for making the SLAC computer facilities available to us.

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